Factors Affecting Density Transients in a Fluidized Bed

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The effects of five operating variables on the uniformity of fluidization in a 4-in.-diameter column were determined by a statistically designed experiment. Density fluctuations inside the bed were measured by a capacitance method. Certain parts of the experiment were repeated in a 24-in.-diameter column for comparison with the 4-in.-column data.

In the 4-in. column gas velocity had by far the greatest effect on uniformity, with uniformity generally decreasing with an increase in gas velocity. This is consistent with the theory that most gas introduced in excess of the rate for incipient fluidization passes through the bed in the form of various-sized bubbles. Better uniformity at the lower bed level indicates that bubbles grow in size as they proceed up the column. An entirely different gas-flow pattern was indicated in the 24-in. column.

Uniform expansion of a fluidized bed with an increase in gas velocity would result in uniform density throughout; this is a condition ideal for maximum solids-gas contact. There are several factors however which favor bubble and agglomerate formation and thus make the bed less than 100% uniform throughout.

If chemical reactions are occurring in the fluidized bed, then uniformity could affect performance through its effect on over-all rate, conversion, or selective reactions if simultaneous reactions can occur. Gas bypassing could result in less over-all contact and conversion. If the reaction mechanism is such that desirable reactions occur at the surface of the solids but undesirable reactions proceed in the gas phase, then bubble formation with its greater gas phase would favor the undesirable reaction. Because of these possible effects of bed uniformity on chemical reactions, the effects of several operating variables on bed uniformity were investigated.

Only a superficial estimate of fluidization uniformity can be made in a transparent or open-top vessel. Quantitative measurements require investigation into the interior of the bed by a measuring device which disturbs the fluidization characteristics to a negligible degree. X-ray absorption (*3*) and electrical-capacitance techniques (*1, 2*) have been used for this purpose. Electrical capacitance was selected for this investigation because it is simpler and because it measures a smaller segment of the bed; it is based on the large difference in dielectric constant between gases and

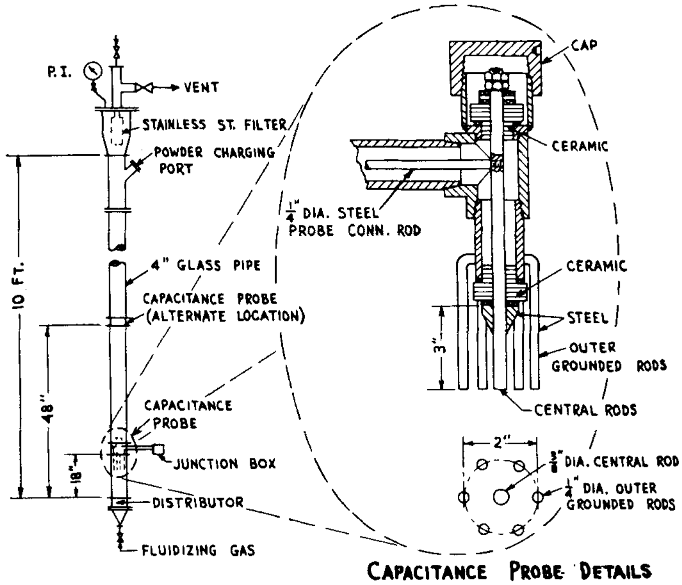

Fig. 1. Four-inch fluidization column.

TABLE 1. SCREEN ANALYSES OF SILICON POWER

	−60-mesh grind			−200-mesh grind	
Mesh size (U.S.S.)	% retained, 4 in. runs	% retained, 24 in. runs	Mesh size (U.S.S.)	% retained, 4 in. runs	% retained, 24 in. runs
50	0	0	50	0	0
70	0.4	0.2	70	0.2	0.2
100	13.3	7.9	100	4.7	0.2
140	17.6	17.3	140	6.7	0.2
200	16.8	18.7	200	5.9	0.4
270	11.8	15.2	270	5.7	10.1
400	16.0	19.7	400	16.7	33.5
−400	24.1	21.0	−400	60.1	55.4
	100.0	100.0		100.0	100.0

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most solids. The presence of a gas-solid mixture (as in a fluidized bed) between the plates of a capacitor produces an effective dielectric constant which is proportional to the relative quantity of each phase. Thus by measuring the capacitance of an appropriately sized probe placed in the bed, one can calculate densities within the probe plates from known dielectric constants of the two phases or from an empirical calibration of density vs. capacitance over the desired density range.

The application of capacitance methods to measurement of fluidization characteristics is not new. It was developed independently at the Bureau of Mines (*1*) and at the duPont laboratories (*2*). New information reported here includes improvements in capacitance measurements and calculations over those reported (*1, 2*) and the results of a statistical experiment designed to evaluate the independent effects and interactions of six operating variables on uniformity of fluidization in a particular system. A probe was designed to give a minimum interference with fluidization in its immediate vicinity and to measure the density of a finite volume. The new method of calculation accounts for the nonlinearity between density and effective dielectric constant. The ranges in bed height and superficial gas velocity have been greatly extended over those reported (*3*).

The results of a statistically designed experiment reported here show the effect of particle size, distributor type, bed level, probe height, superficial velocity, and column diameter. The solid phase consisted of a mixture of metallic silicon and copper powder with a weight ratio of 93/7. Inert gas (approximately 85 to 15 volume ratio of nitrogen and carbon dioxide) was the fluidizing gas. These experiments were conducted at atmospheric pressure and ambient temperature.

APPARATUS AND EXPERIMENTAL PROCEDURES

Capacitance-Measuring Equipment

The only equipment not commercially available was the probe itself; a sketch is shown in Figure 1. The effective portion of the probe was the volume enclosed by the six outer grounded rods, which together with the central rod formed a coaxial capacitance cell 2 in. in diameter and 3 in. long.

TABLE 2. CAPACITANCE-PROBE EXPERIMENT DESIGN

COLUMN DIAMETER →			4 INCH				24 INCH
PROBE HEIGHT ↓	POWDER GRIND ↓	DIST. TYPE →	POROUS CARBON		ORIFICE		ORIFICE
		BED HEIGHT →	5 FT.	9 FT.	5 FT.	9 FT.	9 FT.
18 IN.	-60 MESH	RUN NUMBER →	4-7	4-5	4-6	4-12	24-12
	-200 MESH		4-11	4-9	4-10	4-14	24-14
48 IN.	-60 MESH		4-4	4-8	4-3	4-2	24-2
	-200 MESH		4-15	4-1	4-13	4-16	24-16
SUPERFICIAL VELOCITY, FPS.			0.05 0.1 0.3 0.5 0.7				0.3 0.5 0.7

To compare the uniformity in a 4- and 24-in. column on the same scale, it was necessary to use the same probe size in both columns. It was thought at the time that the maximum bubble size in the 24-in. column was about 3 in. or larger. This probe length was therefore selected, since a completely gas-filled capacitor volume would produce a minimum chart reading. This resulted in a comparatively oversized probe volumetrically in the 4-in. column. It is believed however, on the basis of visual observations, that the seven vertically oriented rods (which occupied only 3% of the column cross-sectional area) had a negligible effect on the fluidization characteristics of the bed in their immediate vicinity and below them. In a separate test, with only the effective portion of the probe (rods) submerged in the bed, the emerging gas bubbles could be observed as they broke the surface in the vicinity of the probe rods. Compared with a probe-free bed no tendency was observed for the gas bubbles to avoid the probe or be broken into smaller bubbles by the probe rods. The author believes that similarly the probe rods affected the fluidization characteristics to a negligible degree when the probe was completely submerged within the bed.

Other capacitance-metering equipment consisted of a detector and a recorder with 0- to 1.0-ma. full-scale sensitivity, 0.2-sec. full-scale response, 1 in./sec. chart speed.

Fluidization Columns

The 4-in. column consisted of a 10-ft.-high section of 4-in. I.D. glass pipe, gas distributor at bottom (described below), a 6-in.-diameter disengaging head at the top of the column; and a star-type stainless steel filter mounted in the disengaging head to prevent powder from leaving the column. The capacitance probe was held in place by a metal spacer sandwiched between sections of the glass pipe at heights of 18 or 48 in. above the gas distributor. Figure 1 shows a sketch of this column.

Two distributor types were used: a 1-in.-thick section of porous carbon and an orifice type having one 0.073-in.-diameter hole every 1.8 sq. in., arranged on 1½-in. equilateral triangle centers.

The 24-in. column was constructed of a 24-in. schedule-20 steel pipe and contained eighteen vertically mounted 1½-in. schedule-40 pipes, arranged on 4½-in. triangular centers (central pipe omitted) to simulate a heat transfer coil. Only the orifice type of distributor was used in this column. Hole size and number per unit of cross-sectional

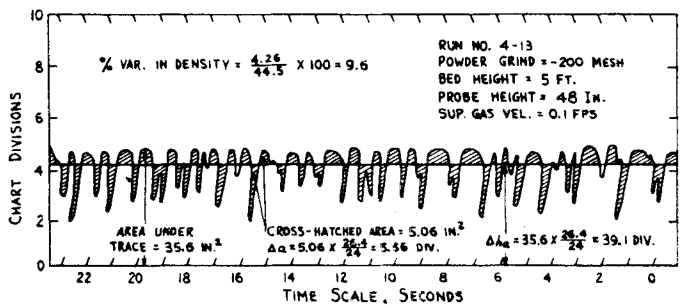

Fig. 2. Typical recorder trace, 4-in. column.

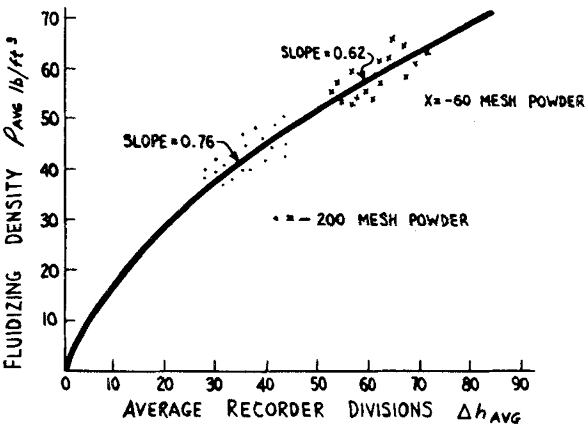

Fig. 3. Fluidization density vs. average recorder deflection.

area were the same as for the 4-in. column; these holes were arranged on $1\frac{1}{8}$-in. square centers.

Powder

Two powder grinds (−60 mesh and −200 mesh of commercial silicon metal) were used, each mixed with 7% by weight of copper powder, 80% of which was below 10 μ.

Screen analyses of the silicon are shown in Table 1.

EXPERIMENTAL PROCEDURE

Fluidizing gas was started through the empty column at a superficial velocity of 0.1 ft./sec. (0.3 ft./sec. in 24-in. column); the capacitance instrument was checked for zero reading. A weighed quantity of powder was charged to the column to the desired bed height. The recorder chart was started and ran for 24 sec. at this gas velocity; bed height was recorded from a visual average of the fluctuating top of the dense phase in the 4-in. column (by Δp measurements in the 24-in. column). The gas velocity was successively adjusted to 0.3, 0.5, 0.7, and 0.05 ft./sec. with recorder-chart traces taken and bed heights observed at each velocity. Velocities were repeated in the same order. Powder was discharged from the column and reweighed. The capacitance probe was again checked for zero reading with the empty column.

The operating conditions were changed as necessary for the next run, and the procedure was repeated.

The same powder charge for each grind was used for all the 4-in. experiments. The 24-in. runs were also made with the same powder charge for each grind, but these were not the same identical charges used in the 4-in. column.

The statistical design is shown in Table 2. Runs were made in the random order shown.

Calculation of Data

Fluidization nonuniformity was calculated in terms of percentage variation in density, defined as (variation in density, lb./cu. ft. × 100)/(average density, lb./cu. ft.) The variation in density represents an integration with respect to time of the irregularity in density above and below the average density. Thus for a perfectly uniform fluidized bed the variations in density above and below the mean and the percentage variation would be zero. The other extreme of 100% variation in density would be represented by the condition of alternately filling the probe volume with all solid and all gas. The values for average density and variation in density were calculated from the recorded values of probe capacitance over a 24-sec. time interval. A typical recorder trace is shown in Figure 2 and will be used to illustrate the method of calculation. The original 24-in. chart length represented a 24-sec. sampling time at a 1-in./sec. chart speed. The average recorder reading was calculated to be 39.1 divisions from a planimeter area of 35.6 sq. in. between the trace and base line, 24-in. chart length, and a conversion factor of 26.4 divisions/in. Average recorder height was drawn on the chart. Area between average line and trace (cross-hatched area) was determined by planimeter (5.06 sq. in.) and converted to equivalent recorder divisions (5.56 divisions). The average recorder height (Δh_a = 39.1) was converted to average density of 44.5 lb./cu. ft. by a calibration chart (Figure 3). This is an empirical calibration chart representing a plot of experimentally determined bed densities (bed weight/measured volume) [illegible] the average recorder reading for all of the 4-in. runs.* The variation in recorder trace (Δa = 5.56 divisions) was converted to variation in density ($\Delta \rho$ = 4.26 lb./cu. ft.) from the slope of the Figure 3 curve (0.76). For simplification in calculation the following conversion factors were used, representing the approximate slopes at the density in question:

−60 mesh = 0.625 lb./cu. ft. per recorder division

−200 mesh = 0.76 lb./cu. ft. per recorder division

Percentage variation in density was then

$$\frac{\text{variation in density, lb./cu. ft.} \times 100}{\text{average density, lb./cu. ft.}} = \frac{4.26}{44.5} \times 100 = 9.6\%$$

Statistical Analysis

The effects of the five operating variables on bed uniformity, plus all interactions, were determined from a statistical analysis of variance. Significant differences between levels were determined by the t-test method. Standard statistical methods were used in this analysis (*6*).

EXPERIMENTAL RESULTS AND DISCUSSION—4-IN.-COLUMN DATA

A comparison of the over-all effects of each of the primary variables may be seen from Table 3, in which averages of percentage variation in density are presented.

Over the ranges investigated, all variables except bed height had a measurable influence on uniformity of fluidization. The relative effect of each of the primary variables was much greater for some than for others, as may be seen from the sums of squares assignable to each primary variable:

Primary variable	Sums of squares	Percentage of total sums of squares due to primary variables
A. Distributor	31.711	0.6
B. Bed height	5.137	0.1
C. Probe height	1,982.253	39.4
D. Powder grind	57.852	1.2
E. Gas velocity	2,959.505	58.8
Total due to primary variables	5,036.458	100.1
A.B.C.D. (within replicates, d.f. = 1)	4.965	
E. (within replicates, d.f. = 80)	86.889	

Gas velocity and probe height had the greatest effects by far, with fluidization uniformity generally being inversely proportional to both of these variables. Powder grind had a detectable but relatively minor effect, with the −200-mesh powder giving better uniformity. The same thing was true of the gas-distributor plate, with the porous type giving slightly better uniformity.

The effects of these variables were far from uniform over the whole experiment, however, as shown by the following table of two- and three-way interactions with superficial velocity, all of which were significant at the 1% level.

Interaction	Sums of squares
1. CE, probe height and superficial velocity	325.8
2. DE, powder size and superficial velocity	209.7
3. BDE, bed height, powder size, and superficial velocity	34.3
4. CDE, probe height, powder size, and superficial velocity	21.3

The only significant two-way interaction exclusive of velocity was between probe height and powder size (interaction CD, with 21.7 sums of squares), which was significant at the 7% level. The direction and magnitude of these interactions with gas velocity may be seen in Figures 4 through 7.*

These data were interpreted as follows:

1. Most fluidizing gas introduced in excess of the rate necessary for incipient fluidization (approximately 0.02 to 0.05 ft./sec.) passes through the powder bed in the form of bubbles. This was indicated by the plateau bed densities at low superficial velocities (Figure 2) as well as by the very large over-all decrease in

*Tabular material has been deposited as document 5873 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $1.25 for photoprints or $1.25 for 35-mm. microfilm.

*See footnote in second column.

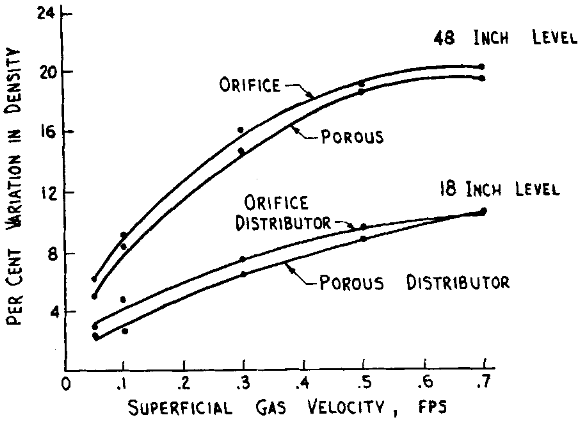

Fig. 4. Interaction among distributor, probe height, and gas velocity in 4-in. column.

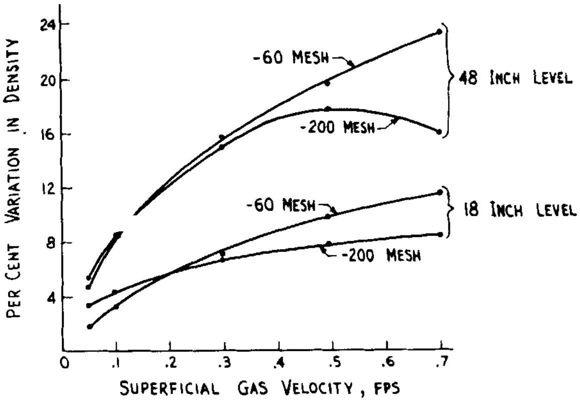

Fig. 6. Interaction among probe height, powder grind, and gas velocity in 4-in. column.

Table 3. Average Percentage Variation in Density for Primary Variables—4-in. Column

Primary variable	Level	Average percentage variation in density	Difference between levels, 1 minus 2
Distributor	A_1 (porous)	9.618	−0.890*
	A_2 (orifice)	10.508	
Bed height, ft.	B_1 (5)	10.242	
	B_2 (9)	9.884	0.358
Probe height, in.	C_1 (18)	6.543	−7.040†
	C_2 (48)	13.583	
Powder grind	D_1 (−60 mesh)	10.664	
	D_2 (−200 mesh)	9.462	1.202*
Velocity, ft./sec.	E_1 (0.05)	4.087	†
	E_2 (0.10)	6.076	
	E_3 (0.30)	11.200	
	E_4 (0.50)	13.942	
	E_5 (0.70)	15.009	

*Represents the significant difference between levels at 5%; that is, if this experiment could be repeated many times (which it cannot), one would expect a difference this large to result from random fluctuations occasionally (less than 5% of the time).
†Represents the significant difference between levels at 1%.

uniformity with an increase in superficial velocity (Table 3). This two-phase system of gas-solid fluidization has been postulated by numerous investigators (*2*, *3*).

2. Gas bubbles grow in size as they proceed up the column, as indicated by the large decay in uniformity between the 18- and 48-in. levels in the column. This is illustrated by the large difference between the 18- and 48-in. average curves in Figure 4. An analogy between bubble coalescence in this and a true liquid-gas system is suggested (*4*).

3. Generally additional powder in the upper part of the column had no effect on the fluidization characteristics in the lower part of the bed, as indicated by the insignificant over-all bed height effect (Table 3). Under the limited conditions of a high gas velocity and fine powder the uniformity was decreased by the addition of four more feet of powder (lower two curves in Figure 5). However under the conditions of high velocity and coarse powder the additional bed height had the opposite effect on fluidization uniformity. Although these peculiar interactions are presently unexplainable, they are believed to be real, as indicated by a significant difference at the 1% level.

4. There was some evidence of an interaction between the type of distributor, probe height, and gas velocity, as indicated by the three-way interaction between these variables significant at the 10% level. The greatest difference between distributors was found at a 0.1 ft./sec. velocity 18 in. above the distributor. (See plot of interaction Figure 4.) These results tend to confirm those of Grohse (*3*), who found a porous type of distributor much superior to an orifice

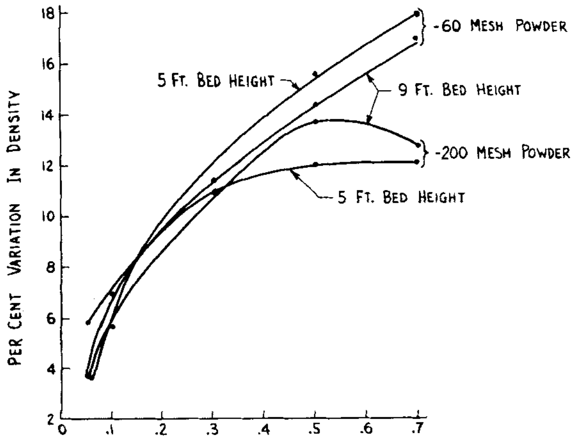

Fig. 5. Interaction among bed height, powder grind, and gas velocity in 4-in. column.

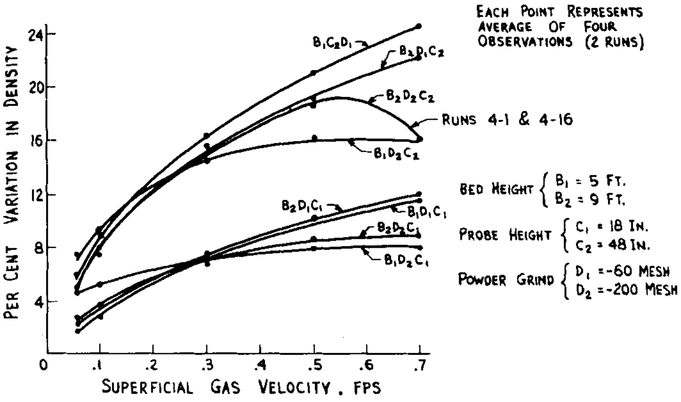

Fig. 7. Interaction among bed height, probe height, powder grind, and gas velocity in 4-in. column.

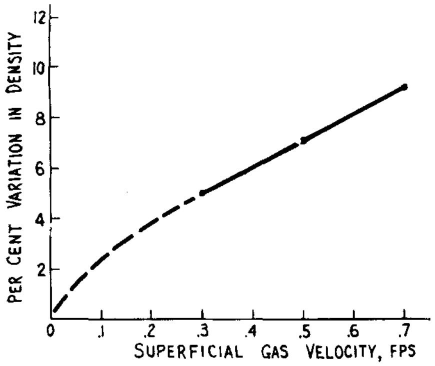

Fig. 8. Variation in density, 24-in. column.

TABLE 6. AVERAGE PERCENTAGE VARIATION IN DENSITY FOR PRIMARY VARIABLES—24-IN. COLUMN

Primary variable	Level	Average percentage variation in density	Difference between levels
Probe height, in.	C_1 (18)	7.13	0.05
	C_2 (48)	7.08	
Powder grind	D_1 (−60 mesh)	7.11	0
	D_2 (−200 mesh)	7.11	
Velocity, ft./sec.	E_3 (0.3)	4.96	
	E_4 (0.5)	7.14	2.18*
	E_5 (0.7)	9.225	2.085*

*Represents the significant difference (by t test) between levels at 5%. The value of the standard error for difference between two means is $(\sigma_{\bar{x}_1} - \bar{x}_2) = 0.989$.

type at most levels in a 15-in.-high bed in the 0.1 ft./sec. velocity region. Morse and Ballou (*2*) reported that the use of a fine-cloth type of distributor improved the uniformity over no distributor for as much as 16 in. above the distributor but that at higher levels the improvement was negligible. It is concluded that outside the narrow limits of 0- to 18-in. bed levels and possibly at gas velocities up to 0.3 ft./sec. the effect of initially good gas distribution is pretty well lost among other more dominant factors.

5. The interaction between powder size and gas velocity (interaction DE) was very pronounced. At the lower gas velocities better fluidization uniformity was obtained with the −60-mesh powder, but at 0.3 ft./sec. and above this trend was reversed. This tendency toward reversal with increasing gas velocity was found at both probe heights but was most pronounced at the 18-in. level (Figure 6, CDE interaction). The poorer fluidization of the −200-mesh powder at low gas velocities has been attributed to gas channeling through the relatively compact bed. This conclusion was confirmed visually. Gas channels, or in some cases distinct bubble paths, were observed under the conditions of low gas velocity and fine powder in the lower section of the column. At higher gas velocities the gas channels broke up, but the trend toward better uniformity was offset by the formation of large bubbles. Bubble formation was found to be favored by the coarse powder, a confirmation of previous findings (*2*).

6. With the coarse powder, bubble growth up the column was found to be accelerated by an increase in gas velocity over the entire range of 0.05 to 0.7 ft./sec. (Figure 6); the same trend was observed for the fine powder up to a velocity of 0.3 ft./sec., but above this velocity bubble growth rate leveled off. An actual increase in uniformity was obtained in some cases by increasing the gas velocity from 0.5 to 0.7 ft./sec. The most general conditions under which a statistically significant decrease was observed were with −200-mesh powder at the 48-in. level, in which the difference in going from 0.5 to 0.7 ft./sec. was 1.6 units. This represents a significant difference, since it is three times the standard error for differences of 0.521 unit. The increase in uniformity in going from 0.5 to 0.7 ft./sec. was even more evident for those runs in which the bed height was 9 ft., used −200-mesh powder, and was at the 48-in. level (runs 4-1 and 4-16). The average of these two runs is shown in Fig. 7 (BCDE interaction). The same trend toward better uniformity was also observed under comparable conditions in the 24-in. column, but the difference between 0.5 and 0.7 ft./sec. was not statistically significant, probably owing to the higher error term in the large column.

At this time no entirely satisfactory explanation can be offered for these unexpected interactions between powder size and gas velocity and for their effect on bubble growth. They may in some way be related to the scale (or eddy size) in turbulent diffusion. This explanation is suggested by a comparison of these data with those reported by Hanratty, *et al.* (*5*) in which the Peclet number was observed to go through a minimum value with an increase in Reynolds number. These phenomena were found to be related more specifically to the scale term in the Peclet number and to the fraction voids. In their liquid-solid (particulate) system the minimum density uniformity occurred at approximately 0.7 fraction void; in the gas-solid system discussed here the minimum density uniformity occurred at approximately 0.75 fraction void. The diffusion phenomena of Hanratty were explained as follows: "Eventually, at a fraction void of 0.70, a fluid element may begin to flow past solid particles without the necessity at each level of flowing laterally in order to evade a particle. Beyond the critical fraction void, in dilute beds, the turbulence is particle generated and the eddy diffusivity then is a direct function of particle population. . . ." Similar phenomena may be occurring in the system presented in this paper, in which at a sufficiently high void volume the tendency for bubbles to move laterally is decreased and their chances of colliding and coalescing with other bubbles thereby reduced. If this explanation is valid, then a similar critical velocity might be expected with the coarse powder at some gas velocity above 0.7 ft./sec., the maximum studied here.

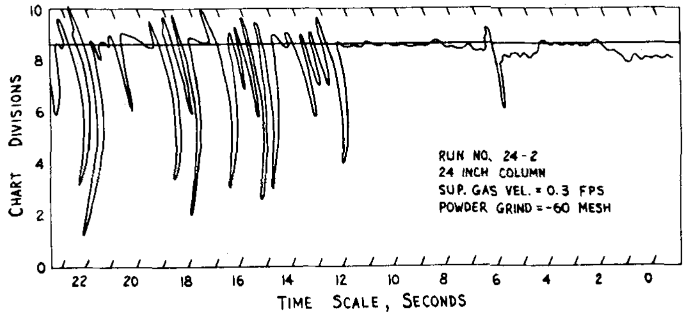

Fig. 9. Typical recorder trace, 24-in. column.

Twenty-four–Inch Results

The average percentage variations in density for the primary variables studied (probe height, powder grind, and gas velocity) are shown in Table 6.* These data were interpreted as follows:

1. In comparison with that of the 4-in. column under comparable conditions the indicated fluidization uniformity was about twice as good in the 24-in. column (7.1 vs. 13.8 over-all).

2. Contrary to the findings in the 4-in. column, no indication of bubble growth was found between the 18- and 48-in. levels in the 24-in. column.

3. Generally the effect of gas velocity was similar in both columns (Figure 8).

*See footnote on page 171.

4. Reproducibility of data was very much poorer in the 24-in. column, especially at the lower velocities.

No entirely satisfactory explanation can be offered to account for all these somewhat surprising results. The author expected larger bubbles in the large column and suspects that large bubbles were actually present but that only occasionally would the bubble paths contact the probe. One reason for believing this is the generally poorer reproducibility of the 24-in. data in comparison with the 4-in. data. Figure 9 shows a good example of this phenomenon. For the first part of the trace practically no bubbles were evident. This was followed immediately by a period in which many large bubbles contacted the probe. Since the probe was located in the horizontal center of the bed, it would have been possible for the bubbles periodically to channel around the probe through other vertical paths. Another reason for suspecting the presence of bubbles in the 24-in. column (but generally undetected by the capacitance probe) is the difference between the over-all density (measured by bed volume and powder weight) and local density at the probe. This difference is in the direction one would expect for such an occurrence. An example of the difference in density is as follows:

Run 24–2	
Powder size:	−60 mesh
Probe height:	48 in.
Superficial gas velocity:	0.5 ft./sec.
Over-all density:	47.5 lb./cu. ft.
Local density at probe:	59.1 lb./cu. ft.

These are believed to be real differences in density. The presence of large bubbles or bubble paths in this column was confirmed subsequently by the helium tracer technique, which showed gross gas bypassing. Because of this nonrepresentative sampling in the center of the bed only, the results obtained in the 24-in. column and reported here should not be used as a true indication of the effect of the primary variables throughout the entire bed.

CONCLUSIONS

4-inch Column

1. Over the ranges studied, all operating variables except bed height had a measurable influence on the uniformity of density throughout the bed.
2. Gas velocity through the powder bed had by far the greatest effect on uniformity, with uniformity generally decreasing with an increase in gas velocity. This was interpreted to mean that most gas introduced in excess of the rate for incipient fluidization passes through the bed in the form of various-sized bubbles.
3. Better uniformity at the lower bed level indicates that bubbles grow in size as they proceed up the column.
4. Bubble growth is accelerated by a coarser powder.
5. Fine powder promotes the formation of gas channels or bubble paths at low gas velocities in the lower section of the powder bed.
6. Except in the vicinity immediately above it the type of distributor has relatively little effect on uniformity.

Twenty-four-inch Column

1. The more uniform density (in comparison with that of the 4-in.) found in the horizontal center of the 24-in. column is not believed to be representative of the entire bed.
2. Gas channels or distinct bubble paths are believed to be present in the column.

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Viscous Flow Relative to Arrays of Cylinders

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The free-surface model, successfully employed to predict sedimentation, resistance to flow, and viscosity in assemblages of spherical particles, has been extended to the case of flow relative to cylinders. It is shown to be in good agreement with existing data on beds of fibers of various types and flow through bundles of heat-exchanger tubes for cases where it can reasonably be expected to apply. Close agreement in the dilute range with the only theoretical treatment for flow parallel to a square array of cylinders provides interesting validation of the model.

The steady slow motion of fluids relative to assemblages of spherical particles has been previously treated mathematically by the use of modification of the unit-cell technique. Results have been reported for sedimentation and resistance to flow (*11*) as well as for viscosity of suspensions (*10*). The present development extends this theory to the case of flow relative to groups of cylindrical objects.

The derivations are developed on the basis that two concentric cylinders can serve as the model for fluid moving through an assemblage of cylinders. The inner cylinder consists of one of the rods in the assemblage and the outer cylinder of a fluid envelope with a free surface. The relative volume of fluid to solid in the cell model is taken to be the same as the relative volume of fluid to solid in the assemblage of cylinders. In effect one assumes that at a distance from the disturbance to fluid motion caused by a cylinder the velocity of flow will not be greatly affected by the exact shape of the outside boundary. The important consideration is that the appropriate boundary condition of no slippage along the walls of the fluid envelope be maintained. The situation is easily visualized in Figure 1, which shows the unit cell in a square array as compared with the model assumed for axial flow. The cross-hatched area occupied by fluid is the same for both the array and model. The dotted line indicates the outside boundary of the fluid envelope at which a condition of no friction is maintained. Employment of appropriate boundary conditions en-